Exam 1 Practice

February 27, 2015



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P(A|B) = P(A)

(a) TRUE(b) FALSE

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.

P(A|B) = P(A) only if A and B are independent. We saw many examples where $P(A|B) \neq P(A)$.



For two independent events A and B,

 $P(A \cap B) = P(A)P(B)$

(a) TRUE(b) FALSE

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If A and B are independent, which by definition means P(A|B) = P(A), then we have $P(A \cap B) = P(A)P(B)$.

This is how we verify if two events are independent: by checking if $P(A \cap B) = P(A)P(B)!$

1 Suppose a family has **two** children.

A= event that the family has at least one boy and at least one girl.

B= the event that the family has at most one girl.

Are A and B independent?

2 Suppose a family has **three** children.

Are A and B (defined exactly as above) independent?

- (a) (1) independent, (2) not independent
- (b) (1) not independent, (2) independent
- (c) (1) and (2) both independent
- (d) (1) and (2) both not independent
- (e) I don't know

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Exercise 3, part (1) Solution

The sample space is $\Omega = \{BB, GG, BG, GB\}$.

Event A (at least one boy and at least one girl) is

$$A = \{BG, GB\}, \text{ thus } P(A) = \frac{|A|}{|\Omega|} = \frac{2}{4} = \frac{1}{2}.$$

Event B (at most one girl) is

$$B = \{BB, BG, GB\}, \text{ thus } P(B) = \frac{|B|}{|\Omega|} = \frac{3}{4}.$$

Now

$$A \cap B = A$$
, thus $P(A \cap B) = \frac{1}{2}$.

So

$$P(A \cap B) \neq P(A)P(B).$$

So A and B are NOT independent.

Exercise 3, part (2) Solution

The sample space is $\Omega = \{BBB, GGG, BGG, GGB, BBG, GBB, GBG, BGB\}.$

Event A (at least one boy and at least one girl) is

 $A = \{BGG, GGB, BBG, GBB, GBG, BGB\}, \text{ thus } P(A) = \frac{|A|}{|\Omega|} = \frac{6}{8} = \frac{3}{4}.$ Event B (at most one girl) is

 $B = \{BBB, BBG, GBB, BGB\}, \text{ thus } P(B) = \frac{|B|}{|\Omega|} = \frac{4}{8} = \frac{1}{2}.$ So $P(A)P(B) = \frac{3}{8}$. Now

$$A \cap B = \{BBG, GBB, BGB\}$$
 thus $P(A \cap B) = \frac{3}{8}$.

So

$$P(A \cap B) = P(A)P(B).$$

So A and B ARE independent.



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Compute the improper integral

$$\int_0^\infty \frac{1}{x^5} dx$$

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(a)
$$=\frac{1}{4}$$

(b)
$$= -\frac{1}{4}$$

$$(c) = 0$$

(e) I don't know.

$$\int_0^\infty \frac{1}{x^5} dx$$

(a)
$$=\frac{1}{4}$$

(b)
$$= -\frac{1}{4}$$

$$(c) = 0$$

(d) diverges

(e) I don't know.

Exercise 4 solution

The integral

$$\int_0^\infty \frac{1}{x^5} dx$$

is improper for 2 reasons: it has an infinite discontinuity at 0 and the upper limit of integration is unbounded. Thus, we need to split it up.

$$\int_0^\infty \frac{1}{x^5} dx = \int_0^1 \frac{1}{x^5} dx + \int_1^\infty \frac{1}{x^5} dx$$

Looking at the first summand,

$$\int_0^1 \frac{1}{x^5} dx = \lim_{b \to 0^+} \int_b^1 \frac{1}{x^5} dx = \lim_{b \to 0^+} \left(-\frac{1}{4} + \frac{1}{4b^4} \right) = \infty.$$

Since the first summand diverges, the whole integral diverges. (Note that the second summand does converge but that doesn't help.)

Write down the form of the partial fraction decomposition of the following rational function. **Do not** solve for the unknown variables, just stop there.

$$\frac{x^4+3}{x(x^2+2)^2(6x^2-x)}.$$

I got terms in my decomposition

(a) 3(b) 4(c) 5

(d) 6 (e) 7

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Exercise 5 Solution

First, we factor the denominator of

$$\frac{x^4+3}{x(x^2+2)^2(6x^2-x)}$$

into linear terms and irreducible quadratics:

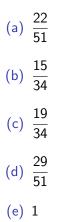
$$x(x^{2}+2)^{2}(6x^{2}-x) = x^{2}(x^{2}+2)^{2}(6x-1),$$

where now all the terms are irreducible. Thus the correct form of the partial fraction decomposition is

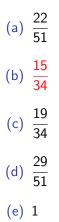
$$\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+2} + \frac{Ex+F}{(x^2+2)^2} + \frac{G}{6x-1}.$$

Two cards are chosen at random from a pack of 52 playing cards. What is the probability that at least one of them is a Heart?

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Exercise 6 Solution

In a pack, there are 13 Hearts and 39 non-Hearts.

Define

A= event that at least one of them is a Heart Therefore, A^c = event that neither of them is a Heart.

Define B= event that first card is not a Heart. So $P(B) = \frac{39}{52} = \frac{3}{4}$.

C = event that second card is not a Heart. So $P(C \mid B) = \frac{38}{51}$

$$\implies P(A^c) = P(B \cap C) = \frac{3}{4} \times \frac{38}{51} = \frac{19}{34}$$

$$\implies P(A) = 1 - P(A^c) = 1 - \frac{19}{34} = \frac{15}{34}$$